

# Tracking Illiquidities in Daily and Intradaily Characteristics<sup>1</sup>

Gulten MERO<sup>2</sup>

co-authors: Serge Darolles<sup>3</sup> and Gaëlle Le Fol<sup>4</sup>

November 25, 2013

---

<sup>2</sup> Université de Cergy-Pontoise and THEMA

<sup>3</sup> Université de Paris-Dauphine and CREST-INSEE

<sup>4</sup> Université de Paris-Dauphine and CREST-INSEE

<sup>1</sup> We gratefully acknowledge financial supports from the chair of the QUANTVALLEY/Risk Foundation: Quantitative Management Initiative, as well as from the project ECONOM&RISK (ANR-2010-blanc 1804 03).

# Motivation

- Return volatility and volume evolutions result from information and liquidity shocks.
  - Information generates trades;
  - Liquidity problems modify the way information is incorporated into price change and volume;
  - On the other hand, information shocks are responsible for the presence of liquidity shocks into the market.
- The interaction between information and liquidity problems can explain some well-known stylized facts.
  - $Cov(R_t, R_{t-1})$  [Getmansky et al. (2004)];
  - $Cov(R_t^2, R_{t-1}^2)$  [GARCH and stochastic volatility models];
  - $Cov(R_t^2, V_t) > 0$  [Andersen (1996), Darolles et al. (2013)...].

# Motivation

- Two aspects of (il)liquidity:
  - Short-term liquidity frictions, due to temporary order imbalances (in the sense of GM), which are resorbed by the market within the trading day and increase the daily traded volume.
  - Time-persistent illiquidity events due to destabilizing margins and volatility spirals (in the sense of Brunneimeier and Pedersen, 2009), provoking the time-persistence of returns, volatility and volume.
- Why is it important to understand liquidity?
  - Detecting investment opportunities for liquidity traders: mean reversion versus momentum strategies exploiting respectively short-term and time-persistent liquidity issues.
  - Regulators must distinguish between both aspects of liquidity and focus on the second one which is inherent to risk that liquidity may disappear from the market resulting in important losses.

# Motivation

## Questions

- How to isolate liquidity problem effects on daily volatility and volume?
- How to separate the respective effects of the two aspects of liquidity?
- How to infer their presence from trading characteristics?

# Main contributions

- We propose a statistic model in order to simultaneously:
  - account for the impact of liquidity frictions on the daily traded volume;
  - account for the time-persistent pattern of liquidity shocks.
- As compared to previous literature, we exploit both data dimensions, time-series and bivariate distribution, and thus exploit both stylized facts,  $Cov(R_t^2, V_t)$  and  $Cov(R_t^2, R_{t-1}^2)$ , in order to:
  - measure the liquidity part of volume;
  - filter time-varying stock-specific liquidity indicators.

# Main results

- Short-term liquidity frictions:
  - impact the traded volume at the intradaily and daily frequencies;
  - affect the stock volatility only at the intradaily frequency.
- The time-persistent liquidity problems:
  - can explain daily volume dynamics;
  - are responsible for stochastic volatility.
- $\Rightarrow$  Filter dynamic and stock-specific liquidity indicator.

# Outline

- 1 Introduction
- 2 Our framework
  - The statistic model
  - Literature review
- 3 The estimation methodology
- 4 Empirical applications
- 5 Concluding remarks

# Outline

- 1 Introduction
- 2 Our framework
  - The statistic model
  - Literature review
- 3 The estimation methodology
- 4 Empirical applications
- 5 Concluding remarks



A bivariate model with two dynamic latent variables, accounting for information and liquidity problems:

$$\begin{aligned}\Delta P_t &= \mu_p I_t^* + \sigma_p \sqrt{I_t^*} Z_{1t}, \\ V_t &= \mu_v^{at} I_t^* + \mu_v^{la} L_t + \sigma_v \sqrt{I_t^*} Z_{2t},\end{aligned}$$

- $I_t^*$  represents the information flow process which is supposed to be time-persistent in order to account for the presence of long-lasting liquidity problems.
- $L_t$  is the latent factor allowing to account for the presence of short-term liquidity frictions which increase the daily traded volume. It is supposed to be serially correlated: in fact, liquidity frictions are not isolated events in time but seem to exhibit time-series clustering.

# The impact of time-persistent liquidity problems on daily price change

- Let  $I_t$  be the iid process of information inflow in the absence of long lasting liquidity problems.
- When liquidity problems persist in time, only part of information hitting the market during the trading day is incorporated in daily price change.
- Let  $x_t$  be the proportion of  $I_t$  incorporated in day  $t$  price change ( $0 < x_t < 1$ );
- Let  $I_t^*$  denote the information process in the presence of long lasting illiquidity events:

$$\begin{aligned} I_t^* &= x_t I_t \\ I_{t+1}^* &= x_{t+1} I_{t+1} + (1 - x_t) I_t. \end{aligned}$$

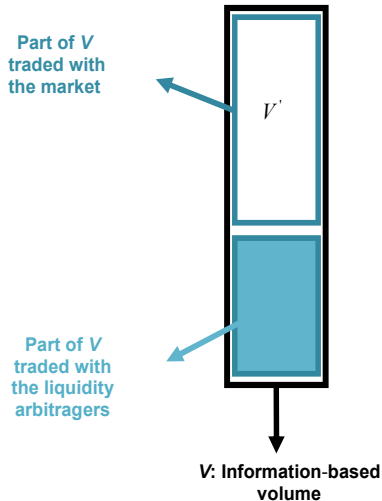
# The impact of liquidity frictions on daily traded volume

- Liquidity is determined by the demand and supply of immediacy;
- A **GM-process** contains only 3 dates: dates 1 and 2 are trading dates, date 3 is used as terminal condition with  $\tilde{P}_3$  being the liquidation value;
- Only 2 market participants:  $J$  active traders (AT) and  $M$  market makers acting as liquidity arbitragers (LA).
- Trade asynchronization at date 1  $\Rightarrow$  Liquidity frictions at date 1  $\Rightarrow$  a temporary order imbalance  $z$ :

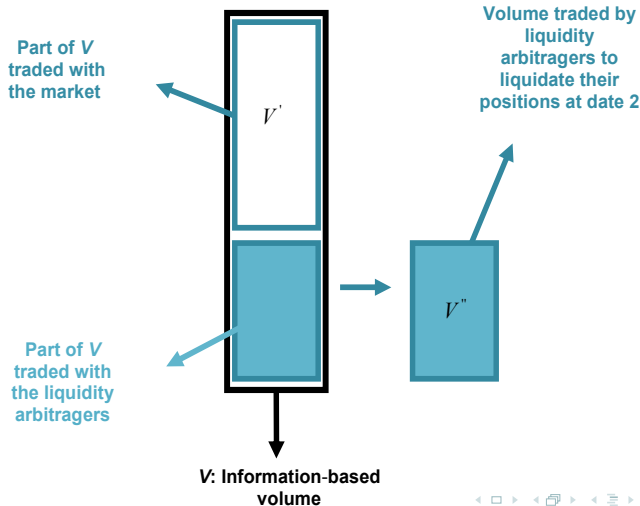
$$z = \sum_{j=1}^{J_1} z_j \neq 0, \quad J_1 < J. \quad (1)$$

- The market makers provide liquidity when needed (date 1) and liquidate their positions at date 2 as other active traders arrive with opposite order imbalances.
- $\Rightarrow$  This increases the total traded volume.

# The impact of liquidity frictions on daily traded volume



# The impact of liquidity frictions on daily traded volume



## Distinguishing the effects of both aspects of liquidity

- In the presence of long-term liquidity problems and short-term liquidity frictions, we have:

$$\begin{aligned}\Delta P_t &= x_t \Delta P'_{1,1} + (1 - x_{t-1}) \Delta P'_{t-1} \\ \Delta P_{t+1} &= x_{t+1} \Delta P'_{t+1} + (1 - x_t) \Delta P'_t.\end{aligned}$$

$$\begin{aligned}V_t &= [x_t V'_t + (1 - x_{t-1}) V'_{t-1}] + V''_t, \\ V_{t+1} &= [x_{t+1} V'_{t+1} + (1 - x_t) V'_t] + V''_{t+1}.\end{aligned}$$

- The triangular structure of our model allows us to distinguish between the two effects of liquidity on the dynamics of the daily traded volume.

- Our bivariate model combines stochastic volatility and states-space formulations for price change and volume respectively;
- $\Rightarrow$  Triangular structure allowing us to:
  - distinguish between both aspects of liquidity;
  - separate information from liquidity shock impacts on daily traded volume
- Model implications: it explains the dynamics of daily trading characteristics:

$$\begin{aligned}
 \text{Cov}(\Delta P_t, \Delta P_{t+1}) &= x_t(1 - x_t)\text{Var}(\Delta P_t') \\
 \text{Cov}(\Delta P_t^2, \Delta P_{t+1}^2) &= x_t^2(1 - x_t)^2\text{Var}((\Delta P_t')^2) \\
 \text{Cov}(V_t, V_{t+1} \mid I_t^*) &= x_t(1 - x_t)\text{Var}(V_t') + \text{Cov}(V_t'', V_{t+1}'').
 \end{aligned}$$

# Outline

- 1 Introduction
- 2 Our framework
  - The statistic model
  - Literature review
- 3 The estimation methodology
- 4 Empirical applications
- 5 Concluding remarks



## Empirical research: positive volatility-volume relationship

- Clark (1976), Epps and Epps (1976), Copeland (1976-77), Tauchen and Pitts (1983), Harris (1983-86).

Theoretical explanation comes from microstructure models: Information  $\Rightarrow$  positive volatility-volume relation.

- Kyle(1986), Glosten and Milgrom (1985), Easley and O'Hara (1987), Easley et al.(1996).

Mixture of Distribution Hypothesis (MDH) explores the microstructure framework:

- Tauchen and Pitts (1983), Harris (1983-86), Richardson and Smith (1994), Andersen (1996).

# The standard MDH model of Tauchen and Pitts (1983)

- Information is responsible for return and volume evolutions;
- Static framework.
- The market is perfectly liquid.

$$\Delta P_t = \sum_{i=1}^{I_t} \Delta P_i, \quad \Delta P_i \sim N(0, \sigma_p^2) \quad \Leftrightarrow \quad \Delta P_t = \sigma_p \sqrt{I_t} Z_{1t}$$

$$V_t = \sum_{i=1}^{I_t} V_i, \quad V_i \sim N(\mu_v, \sigma_v^2) \quad \Leftrightarrow \quad V_t = \mu_v I_t + \sigma_v \sqrt{I_t} Z_{2t}$$

where  $Z_{1t}$  and  $Z_{2t}$  are i.i.d. standard normals and independent of  $I_t$ .

- The volatility and volume are positively correlated:

$$\text{Cov}(\Delta P_t^2, V_t) = \sigma_p^2 \mu_v \text{Var}[I_t] > 0$$

# Richardson and Smith (1994) model specification

- Include a mean parameter in the price change equation;
- Static framework.
- The market is perfectly liquid.

$$\Delta P_t = \mu_p I_t + \sigma_p \sqrt{I_t} Z_{1t} \quad (2)$$

$$V_t = \mu_v I_t + \sigma_v \sqrt{I_t} Z_{2t} \quad (3)$$

# Our contribution in the literature

- Our model can be considered as a statistic extension of Richardson and Smith (1994) model toward two directions:
  - Measuring the liquidity part of volume by adding a second latent variable in the volume equation based on GM definition of liquidity;
  - Extending the return equation in order to capture the time-persistence pattern of liquidity by proposing a liquidity-based interpretation of stochastic volatility.

## Step 1: Stochastic volatility formulation for $\Delta P_t$ equation

$$\Delta P_t = \mu_p I_t^* + \sigma_p \sqrt{I_t^*} Z_{1t}, \quad (4)$$

$$\ln I_t^* = \beta \ln I_{t-1}^* + \eta_t. \quad (5)$$

- Long-lasting liquidity problem interpretation of stochastic volatility effect;
- Markov regime switching techniques to estimate (4)-(5) and filter  $I_t^*$  [Hwang, Satchell and Pereira (2007)].
- The standard SV model yields extremely high levels of persistence; Allowing for regime switching in the level of volatility reduces the considerably reduces the persistence parameters.

## Step 2: State space formulation for $V_t$ equation

$$V_t = \mu_v^{at} I_t^* + \mu_v^{la} L_t + \sigma_v \sqrt{I_t^*} Z_{2t}, \quad (6)$$

$$L_t = aL_{t-1} + \omega_t. \quad (7)$$

- $I_t^*$  is replaced by the one filtered in step 1;
- Kalman filter algorithm to estimate (6)-(7) and filter  $L_t$ .
- This specification nests that of Hamilton with iid  $L_t$  as a special case.

# The data

- Individual stocks belonging to FTSE100;
- Daily return and turnover time series.

## Some empirical results

### Long lasting illiquidity events and momentum strategies

- Parameters of interest:  $\mu_p$  and  $\beta$ ;
- Implications for momentum trading strategies;
- $(\mu_p, \beta)$  versus sample serial correlation coefficients;

### Short-term liquidity frictions and high frequency liquidity arbitrage

- Parameters of interest:  $\mu_v^{la}$  and  $a$ ;
- Implications for intraday liquidity arbitrage strategies;
- Immediacy cost.

### Filtering dynamic liquidity indicators



# Illiquidity events and momentum strategies (1)

Panel A:  $\hat{\rho}_{(R_t, R_{t-1})} = 0$  and  $\hat{\rho}_{(R_t^2, R_{t-1}^2)} > 0$

ID	$\mu_p$	$\mu_0$	$\mu_1$	$\beta$	$\sigma_{\eta,0}$	$\sigma_{\eta,1}$	$\sigma_\phi$
PSN	0,0004**	-12,14**	-8,84**	0,15**	2,3598**	1,4004**	-0,0003
RR	0,0012**	-14,65**	-9,43**	0,92**	0,4737*	0,1620	1,4554**
SGE	0,0002	-12,76**	-9,20**	0,83**	0,4106	0,4140**	1,1200**
SGRO	0,0001	-11,83**	-9,28**	0,17**	2,4783**	1,2646**	-0,0001
XTA	0,0003	-12,92**	-8,58**	0,04	3,3120**	1,6084	-0,0001

Panel B:  $\hat{\rho}_{(R_t, R_{t-1})} = 0$  and  $\hat{\rho}_{(R_t^2, R_{t-1}^2)} = 0$

ID	$\mu_p$	$\mu_0$	$\mu_1$	$\beta$	$\sigma_{\eta,0}$	$\sigma_{\eta,1}$	$\sigma_\phi$
CNA	0,0008**	-17,35**	-9,65**	0,90**	0,3679	-0,1756**	-1,5422**
ITV	0,0006**	-13,75**	-9,22**	0,12**	-1,5672**	1,3934**	0,0060
IVZ	0,0002	-9,13**	-14,84**	0,06	1,6622**	4,0987**	0,0004
RDSB	0,0002	-9,75**	-13,23**	0,06	-1,4248**	2,9780**	0,2681

## Illiquidity events and momentum strategies (2)

- The empirical first-order serial correlation of returns is not a sufficient criteria to select stocks to be included in momentum strategies;
- The tests of significance of the sample autocorrelation coefficients are not appropriate since they don't account for volatility clustering.
- A stock may have sample autocorrelations not significantly different from zero when performing classical test statistics and still be affected by long-term liquidity problems whose presence can be empirically inferred using our model.
- In particular, according to our framework, the long-lasting liquidity problems result in  $\mu_p$  and  $\beta$  parameters statistically positive.
- For example, according to the serial correlation criteria, only stocks 6, 18 and 19 should be included in the momentum strategies; our approach allows us to select somme additional stocks (59, 68, 20 and 41).

# Liquidity frictions and high frequency liquidity arbitrage (1)

ID	$\mu_v^{at}$	$\mu_v^{la}$	$\sigma_v$	$a$	$\sigma_w$
PSN	0,0005	0,0016**	0,0026**	0,97**	0,81**
RR	0,0085	0,0022**	0,0041**	0,90**	0,66
SGE	0,0030	0,0038	0,0031**	0,97**	0,15
SGRO	0,0001	0,0060**	0,0020**	0,94**	0,15**
XTA	0,0013	0,0662	0,0098**	0,95**	0,02
CNA	0,0064	0,0014	0,0029**	0,70**	1,71
ITV	0,0009	0,0039**	0,0040*	0,93**	0,96
IVZ	0,0003	0,0038	0,0016**	0,90**	1,29
RDSB	0,0001	0,0014*	0,0003**	0,90**	0,33

## Liquidity frictions and high frequency liquidity arbitrage (2)

- Once  $I_t^*$  filtered, we can filter  $L_t$  conditional on  $I_t^*$  using the volume equation.
- The parameters of interest here is  $\mu_V^{la}$  which allows us to identify stocks that are subject to short-term liquidity frictions.
- These stocks represent liquidity arbitrage opportunities at the intradaily frequency.
- These investment opportunities are a source of trade for liquidity arbitragers who enter the market to provide the missing liquidity and liquidate their positions in order to cash the liquidity premium.

# Subperiod analysis (1)

January 2005 - June 2007

ID	$\mu_p$	$\mu_0$	$\mu_1$	$\beta$	$\sigma_{\eta,0}$	$\sigma_{\eta,1}$	$\sigma_\phi$
ABF	0,0004**	-16,46**	-10,49**	0,75**	0,0001	0,4692**	1,5767**
ATST	0,0052**	-10,95**	-8,36**	0,90**	0,2603**	0,0000	2,0937**
ANTO	0,0002	-11,03**	-8,32**	0,05	2,7315**	1,3141**	0,0002
BG	0,0013**	-14,07**	-9,18**	0,91**	0,3175	0,1992**	1,3930**
BLND	0,0010**	-11,38**	-8,77**	0,24**	1,8722**	0,4584	0,9803**

July 2007 - May 2009

ID	$\mu_p$	$\mu_0$	$\mu_1$	$\beta$	$\sigma_{\eta,0}$	$\sigma_{\eta,1}$	$\sigma_\phi$
ABF	0,0004	-12,33**	-8,77**	-0,16	1,9633**	0,0004	1,5765**
ATST	0,0003**	-10,68**	-8,83**	0,99**	4,3015**	0,2347**	1,8850**
ANTO	0,0006**	-6,90**	-10,75**	0,98**	0,1332**	0,0004	1,4705**
BG	0,0003**	-12,33**	-7,99**	0,98**	0,5118**	0,0000	1,3410**
BLND	0,0009**	-33,80**	-8,03**	0,88**	0,8095**	0,4062**	1,9187**

## Subperiod analysis (2)

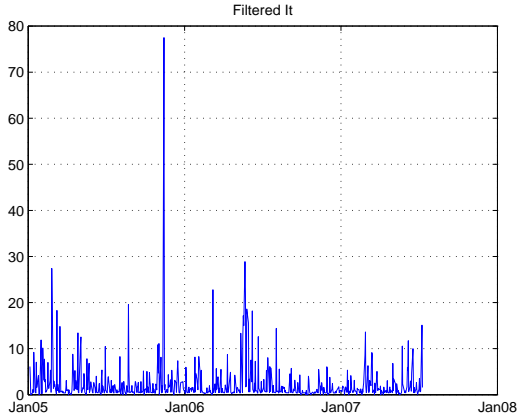
January 2005 - June 2007

ID	$\mu_v^{at}$	$\mu_v^{la}$	$\sigma_v$	$a$	$\sigma_w$
ABF	0,0022**	0,0043**	0,0014**	0,90**	0,1203**
ATST	0,0044**	0,0046	0,0060**	0,98**	0,2987**
ANTO	0,0005**	0,0155**	0,0050**	0,91**	0,6301**
BG	0,0040**	0,0056**	0,0198**	0,89**	0,1200**
BLND	0,0001**	0,0165**	0,0014**	0,87	1,3009**

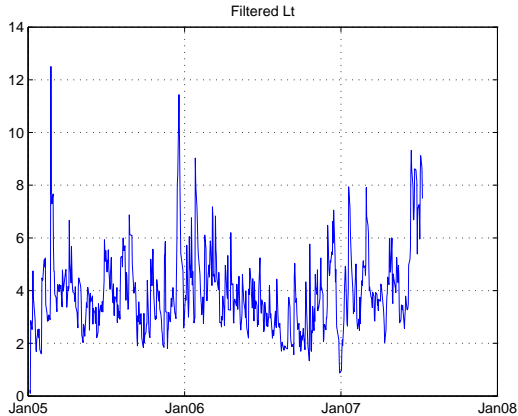
July 2007 - May 2009

ID	$\mu_v^{at}$	$\mu_v^{la}$	$\sigma_v$	$a$	$\sigma_w$
ABF	0,0002**	0,0036**	0,0010**	0,99**	0,2078**
ATST	0,0009**	0,0028**	0,0001	0,49	1,0500
ANTO	0,0041**	0,0037**	0,0002	0,96**	0,2078**
BG	0,0014**	0,0072**	0,0006**	0,90**	0,1801**
BLND	0,0038**	0,0085**	0,0043*	0,98**	0,1962**

# Persimmon Plc (stock 59)

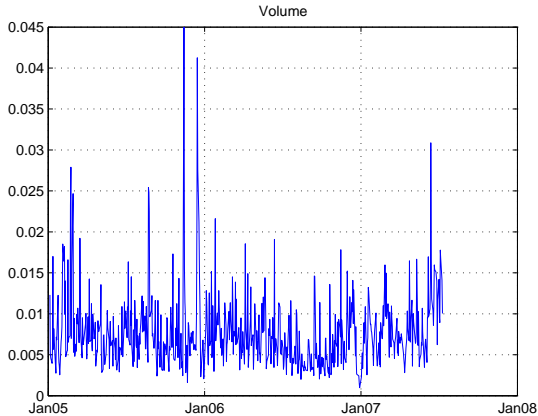


# Persimmon Plc (stock 59)





# Persimmon Plc (stock 59)



## Paper Contributions:

- Short-term liquidity frictions and long lasting illiquidity events have not the same impact on daily returns and volume;
- Decomposing the daily traded volume into two components due to information and liquidity.
- Extracting dynamic stock-specific liquidity indicators.

## Further research

- Confront our liquidity indicators to liquidity microstructure measures;
- Empirical tests of the validity of our liquidity measure;
- Build up market liquidity indicators;
- Cross-sectional factor analysis to capture the essence of commonalities in liquidity shocks.