When Market Illiquidity Generates Volume

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Introduction

The standard MDH model The Grossman and Miller (1988) model Our framework Empirical applications Concluding remarks

Motivation

- Modeling the price and volume evolutions.
 - Intradaily;
 - Daily.
- Understanding price volatility and volume.
 - Information;
 - (II)liquidity.
- 2 trading motives: information-based and liquidity-based trading.

Questions

- Which type of trade is a response to information versus liquidity shocks?
- What volume does not tell?
- How to filter information and liquidity shocks from trading characteristics?

Main contributions

We model their respective impacts on daily price change and trading volume based on:

- the contemporaneous relation between daily returns and volume;
- the microstructure model of Grossman and Miller (1988);
- the mixture of distribution hypothesis (MDH) model of Tauchen and Pitts (1983).

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The MDH model

The information flow is the only source of the positive correlation volatility-volume.



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- the contemporaneous relation between daily returns and volume;
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- the mixture of distribution hypothesis (MDH) model of Tauchen and Pitts (1983).

The MDHL model

Modified MDH model (MDHL) accounting for both information and liquidity shocks.



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Main contributions (continued)

New results

- Information shocks impact price variations and volume at any frequency;
- Liquidity shocks:
 - impact intradaily volume and price variations;
 - increase daily volume;
 - have no impact on daily price variations.

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- Information shocks impact price variations and volume at any frequency;
- Liquidity shocks:
 - impact intradaily volume and price variations;
 - increase daily volume;
 - have no impact on daily price variations.

New liquidity indicator

- Static and stock-specific liquidity measure extracted from daily returns and volume;
- Identify stocks affected by liquidity frictions during a given period.

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The MDH framework The Tauchen and Pitts (1983) model

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- The Grossman and Miller (1988) model

Our framework

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- Generalization
- The MDHL model
- 5 Empirical applications
 - GMM estimations
 - Empirical results

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Empirical research: positive volatility-volume relationship

Clark (1976), Epps and Epps (1976), Copeland (1976-77), Tauchen and Pitts (1983), Harris (1983-86).

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• Kyle(1986), Glosten and Milgrom (1985), Easley and O'Hara (1987), Easley et al.(1996).

Mixture of Distribution Hypothesis (MDH) explores the microstructure framework:

• Tauchen and Pitts (1983), Harris (1983-86), Richardson and Smith (1994), Andersen (1996).

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Main assumptions

- Information drives the positive volatility-volume relationship;
- The movement from one within-day equilibrium to the next is initiated by the arrival of new information to the market;
- The number of within-day t equilibria, I_t , is random.

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The MDH framework The Tauchen and Pitts (1983) model

• Active traders revise their reservation price:

 $\Delta P_{ij}^* = \phi_i + \psi_{ij},$

where ϕ_i and ψ_{ij} are i.i.d. and mutually independent: $\phi_i \sim N(0, \sigma_{\phi}^2), \ \psi_{ij} \sim N(0, \sigma_{\psi}^2)$.

Interpretation:

- ϕ_i is common to all traders and depend on the information,
- ψ_{ij} is specific to trader *j* and sketches the traders profile.

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- Interpretation:
 - ϕ_i is common to all traders and depend on the information,
 - ψ_{ij} is specific to trader j and sketches the traders profile.
- The *i*th equilibrium price change and volume can then be written:

$$\Delta P_i = \frac{1}{J} \sum_{j=1}^J \Delta P_{ij}^* = \phi_i + \bar{\psi}_i, \qquad (1)$$

$$V_{i} = \frac{\alpha}{2} \sum_{j=1}^{J} |\Delta P_{ij}^{*} - \Delta P_{i}| = \frac{\alpha}{2} \sum_{j=1}^{J} |\psi_{ij} - \bar{\psi}_{i}|.$$
(2)

where J is the number of active traders and $\bar{\psi}_i = \frac{1}{J} \sum_{j=1}^J \psi_{ij}$.

The MDH framework The Tauchen and Pitts (1983) model

Conditional on the mixing variable I_t , the bivariate normal mixture is:

$$egin{aligned} \Delta P_t &= \sum_{i=1}^{l_t} \Delta P_i, \qquad \Delta P_i \sim \mathcal{N}(0,\sigma_p^2) \ V_t &= \sum_{i=1}^{l_t} V_i, \qquad V_i \sim \mathcal{N}(\mu_v,\sigma_v^2) \end{aligned}$$

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$$\begin{split} \Delta P_t &= \sum_{i=1}^{l_t} \Delta P_i, \qquad \Delta P_i \sim \mathcal{N}(0, \sigma_p^2) \qquad \Leftrightarrow \quad \Delta P_t = \sigma_p \sqrt{l_t} Z_{1t} \\ V_t &= \sum_{i=1}^{l_t} V_i, \qquad V_i \sim \mathcal{N}(\mu_v, \sigma_v^2) \qquad \Leftrightarrow \quad V_t = \mu_v l_t + \sigma_v \sqrt{l_t} Z_{2t} \end{split}$$

where Z_{1t} and Z_{2t} are i.i.d. standard normals and independent of I_t , while $(\sigma_p^2, \mu_v, \sigma_v^2)$ are functions of $(\sigma_{\phi}^2, \sigma_{\psi}^2)$.

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Volatility-volume relation

The volatility and volume are positively correlated:

$$Cov(\Delta P_t^2, V_t) = \sigma_p^2 \mu_v Var[I_t] > 0$$

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Conclusion

According to the standard MDH model:

- The information flow is responsible for volatility and volume variability;
- The liquidity frictions are ignored.

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- Liquidity is determined by the demand and supply of immediacy;
- A GM-process contains only 3 dates: dates 1 and 2 are trading dates, date 3 is used as terminal condition with P
 ₃ being the liquidation value;
- Only 2 market participants: J active traders (AT) and M liquidity arbitragers (LA).
- Trade asynchronization at date 1 ⇒ Liquidity frictions at date 1 ⇒ a temporary order imbalance z:

$$z = \sum_{j=1}^{J_1} z_j \neq 0, \quad J_1 < J.$$
 (3)

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• When $J_1 = J$, z vanishes.

Date 1 of the GM-process

- J_1 out of J active traders $\Rightarrow z \neq 0 \Rightarrow M$ liquidity arbitragers;
- Maximizing the expected utility of terminal wealth \Rightarrow trader's excess demands, $Q_1^{at_1}$ (aggregated excess demand of AT) and Q_1^{la} (excess demand per LA);
- Equilibrium requires: $Q_1^{at_1} + MQ_1^{la} = 0$,

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$$P_1 = E_1 \tilde{P}_3 - \frac{z \alpha Var_1(E_2 \tilde{P}_3)}{1+M}, \qquad (4)$$

where: $P'_1 = E_1 \tilde{P}_3$ is the price response to new information and $P''_1 = -\frac{z\alpha Var_1(E_2 \tilde{P}_3)}{1+M}$ is the price distortion due to liquidity shock.

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Trader's excess demands become:

$$Q_1^{at_1} = -\frac{M}{1+M}z, \quad MQ_1^{la} = \frac{M}{1+M}z.$$
 (5)

• The LA face a participation cost $c > 0 \Rightarrow$ finite $M \Rightarrow$ limited immediacy providing capacity and $P_1 \neq E_1 \tilde{P}_3$.

Date 2 of the GM-process

Let,

- $Q_2^{at_1}$ be the aggregated excess demand at date 2 of AT who arrived at date 1;
- Q_2^{la} be the excess demand per LA in order to liquidate their positions at date 2;
- Q₂^{at₂} be the aggregated excess demand of AT who arrive at date 2 with opposite order imbalance.

Equilibrium requires: $Q_2^{at_1} + MQ_1^{l_a} + Q_2^{at_2} = 0.$

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Equilibrium requires: $Q_2^{at_1} + MQ_1^{la} + Q_2^{at_2} = 0.$

It follows that, the equilibrium price at date 2 is:

$$P_2 = E_2 \tilde{P}_3, \tag{6}$$

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It follows that, the equilibrium price at date 2 is:

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Trader's excess demands become:

$$Q_2^{at_1} = -z, \quad Q_2^{at_2} = z, \quad Q_2^{la} = 0.$$
 (7)

• The GM framework focuses on the consequences of an order imbalance on the intraday patterns of price change and transaction volume.

GM model implications at the intradaily frequency

In the presence of liquidity frictions and exogenous participation costs for the LA:

- the traded volume at date 1 is lower than it would have been if there were no order imbalance |Q₁^{at₁}| < |z|;
- the transaction price at date 1 deviates from its revealing information level (P₁ ≠ E₁ P̃₃).

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Extending the GM model at the daily frequency $\ensuremath{\mathsf{Generalization}}$ The MDHL model

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Order imbalances and the total price change



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Order imbalances and the total price change



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Order imbalances and the total traded volume



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Order imbalances and the total traded volume



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Implications at the daily frequency

- Illiquidity (order imbalance) has no effect on the total price change. The price change is only due to the information flow;
- Illiquidity (order imbalance) increases the total traded volume.

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- Each trading day is considered as a sequence of 2-date GM-processes.
- The terminal condition date is reported at the end of the trading day; the liquidation value of the risky asset is P

 _T.
- During a trading day t there are i ($i = 1, ..., I_t$) information arrivals and I ($I = 1, ..., L_t$) liquidity frictions. I_t and L_t are random.
- *L_t* is given by:

$$L_t = \sum_{i=1}^{l_t} \delta_i,\tag{8}$$

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where $\delta_i = 1$ if a liquidity friction occurs at date *i* and 0 otherwise $[\delta_i \sim iid B(p)]$.

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Implications concerning daily stock characteristics

• Summing the $\Delta P'_i$, $\Delta P''_i$ and $-\Delta P''_i$, yields the day t price change ΔP_t :

$$\Delta P_t = \sum_{i=1}^{l_t} \Delta P'_i + \sum_{i=1}^{l_t} \delta_i \Delta P''_i - \sum_{i=1}^{l_t} \delta_i \Delta P''_i = \sum_{i=1}^{l_t} \Delta P'_i.$$
(9)

• Summing the $V_i^{'}$ and $V_i^{''}$, yields the day t traded volume V_t :

$$V_t = \sum_{i=1}^{l_t} V'_i + \sum_{i=1}^{l_t} \delta_i V''_i.$$
 (10)

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- Develop an econometric framework to estimate the impact of liquidity frictions on daily stock characteristics:
 - the MDH model is a reduced econometric econometric form of microstructure framework [e.g., Glosten and Milgrom (1985)];
 - we develop a modified MDH model, the MDHL model, which helps infer the presence of liquidity frictions from daily price change and volume time series.

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Daily price change distribution

• The daily price change is not affected by the presence of liquidity frictions.

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Daily price change distribution

- The daily price change is not affected by the presence of liquidity frictions.
- As in TP, the intradaily price increment is normally distributed with mean zero and variance σ²_ρ, which yields:

$$\Delta P_t = \sum_{i=1}^{l_t} \Delta P'_i, \qquad \Delta P'_i \sim N(0, \sigma_p^2) \quad \Leftrightarrow \quad \Delta P_t = \sigma_p \sqrt{l_t} Z_{1t}. \tag{11}$$

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Daily volume distribution (1)

• Let *V_i* be the cumulated traded volume across dates 1 and 2 due to the *i*th piece of information and the liquidity event occurring to the *i*th equilibrium:

$$V_i = V_i^{\prime} + V_i^{\prime\prime},$$

where

- $V'_i = \frac{1}{2} \sum_{j=1}^{J} |z_{ij}|$ is the traded volume due to the *i*th information arrival to the market at day t;
- $V''_i = \frac{M}{1+M} \mid z_i \mid$ is the traded volume due to the intervention of the LA.
- If all the active traders were present in the market after the *i*th information arrival, $V_i = V'_i$.

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Daily volume distribution (2)

• The volume component V'_i due to information is the same as in TP: $V^{TP}_i = \frac{\alpha}{2} \sum_{i=1}^{J} |\psi_{ij} - \bar{\psi}_i|$, with $\psi_{ij} \sim N(0, \sigma_{\psi}^2)$.

$$V_{i}^{'} = \frac{1}{2} \sum_{i=1}^{J} |z_{ij}| = \frac{\alpha}{2} \sum_{i=1}^{J} |\psi_{ij} - \bar{\psi}_{i}| = V_{i}^{TP}.$$
 (12)

$$z_i = \sum_{j=1}^{J_1} z_{ij} = \alpha \sum_{j=1}^{J_1} (\psi_{ij} - \bar{\psi}_i).$$
(13)

It follows that:

$$\sigma_z^2 = \alpha^2 J_1^2 \left(\frac{J - J_1}{J J_1}\right) \sigma_{\psi}^2. \tag{14}$$

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Daily volume distribution (3)

• Intraday price variation due to liquidity frictions, P_i'' , is:

$$\Delta P_{i}^{\prime\prime} = (\Delta P_{i} - \Delta P_{i}^{\prime}) = \frac{1}{J_{1}} \sum_{j=1}^{J_{1}} \Delta P_{ij}^{*} - \frac{1}{J} \sum_{j=1}^{J} \Delta P_{ij}^{*},$$
(15)

where, from TP, $\Delta P_{ij}^* = \phi_i + \psi_{ij}$.

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Daily volume distribution (3)

• Intraday price variation due to liquidity frictions, P_i'' , is:

$$\Delta P_{i}^{''} = (\Delta P_{i} - \Delta P_{i}^{'}) = \frac{1}{J_{1}} \sum_{j=1}^{J_{1}} \Delta P_{ij}^{*} - \frac{1}{J} \sum_{j=1}^{J} \Delta P_{ij}^{*},$$
(15)

where, from TP, $\Delta P_{ij}^* = \phi_i + \psi_{ij}$.

It can be shown that:

$$\Delta P_{i}^{''} = \frac{1}{J_{1}} \sum_{j=1}^{J_{1}} (\psi_{ij} - \bar{\psi}_{i}), \qquad (16)$$

which yields:

$$z_i = \alpha J_1 \Delta P_i^{\prime\prime} \Rightarrow V_i^{\prime\prime} = a \mid \Delta P_i^{\prime\prime} \mid, \tag{17}$$

where $a = \alpha \frac{M}{1+M} J_1$.

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Daily volume distribution (4)

• The daily traded volume, V_t , as a function of L_t :

$$V_t = \sum_{i=1}^{l_t} V'_i + \sum_{i=1}^{l_t} \delta_i V''_i,$$

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Daily volume distribution (4)

• The daily traded volume, V_t , as a function of L_t :

$$V_t = \sum_{i=1}^{l_t} V'_i + \sum_{i=1}^{l_t} \delta_i V''_i,$$
$$V_t = \sum_{i=1}^{l_t} V'_i + \sum_{k=1}^{L_t} V''_k,$$

where $k = 1, ..., L_t$ is a subsequence of $i = 1, ..., I_t$ such as $\delta_i = 1$.

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The MDHL model

• Conditional on I_t and L_t , the bivariate normal mixture is:

$$\Delta P_t = \sigma_p \sqrt{I_t} Z_{1t}, \qquad (18)$$

$$V_t = \mu_v^{at} I_t + \mu_v^{la} L_t + \sigma_v \sqrt{I_t} Z_{2t}.$$
(19)

•
$$Cov(I_t, L_t) = pVar(I_t).$$

• The volatility-volume relationship is:

$$Cov(\Delta P_t^2, V_t) = \sigma_p^2(\mu_v^{at} + p\mu_v^{la}) Var(I_t).$$
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GMM estimations Empirical results

The data:

- Individual stocks belonging to FTSE100;
- Daily return and turnover time series.

GMM tests [Hansen (1982) and Newey and West (1987)]:

- Compute the unconditional 2nd, 3rd, 4th and cross moments of returns and volume implied by the MDHL model;
- The unconditional moments are function of the model parameters, θ ;
- $\theta = (\mu_v^{at}, \mu_v^{la}, \sigma_p^2, \sigma_v^2, m_{2I}, p);$
- Normalization: $E[I_t] = 1 \Rightarrow E[L_t] = pE[I_t] = p$.

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GMM estimations Empirical results

The moment condition vector:

$$g_{T}(\theta) = \frac{1}{T} \sum_{t=1}^{T} \begin{pmatrix} (V_{t} - E(V_{t})) & (1) \\ (R_{t} - E(R_{t}))^{2} & (2) \\ (V_{t} - E(V_{t}))^{2} & (3) \\ (R_{t}^{2} - E(R_{t}^{2}))(V_{t} - E(V_{t})) & (4) \\ (R_{t}^{2} - E(R_{t}^{2}))(V_{t}^{2} - E(V_{t}^{2})) & (5) \\ (V_{t} - E(V_{t}))^{3} & (6) \\ (R_{t} - E(R_{t}))^{4} & (7) \\ (V_{t} - E(V_{t}))^{4} & (8) \\ (R_{t} - E(R_{t}))^{2}(V_{t} - E(V_{t}))^{2} & (9) \end{pmatrix}$$

- 9 moment conditions and 6 parameters \Rightarrow 3 overidentifying restrictions;
- Newey and West (1987) methodology for the weighting matrix.

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GMM estimations Empirical results

Outline

- Introduction
- 2 The standard MDH mode
 - The MDH framework
 - The Tauchen and Pitts (1983) model
- 3 The Grossman and Miller (1988) mode
- Our framework
 - Extending the GM model at the daily frequency
 - Generalization
 - The MDHL model

5 Empirical applications

- GMM estimations
- Empirical results

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GMM estimations Empirical results

- Global validity: χ^2_3 statistic
 - The MDHL model is accepted by the data for 83% of stocks.
- Parameter significance:
 - 39 stocks are concerned by liquidity frictions;
 - μ_v^{at} is significant for 84 stocks and $p\mu_v^{la}$ for 39;
 - If $\mu_v^{\prime a} = 0 \Rightarrow$ standard MDH;
 - μ_v^{at} and $p\mu_v^{la}$ interpretation...

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- Some illustrative examples:
 - (μ^{at}_ν + pμ^{la}_ν) = 0.00624, for Associated British Foods (stock 2);
 - $(\mu_v^{at} + p \mu_v^{la}) = 0.00589$,

for Barclays PLC (stock 8);

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Paper contributions

- Extension of GM microstructure model at the daily frequency ⇒ new structural model allowing us to model the impact of liquidity frictions on daily data;
- Extension of the MDH framework ⇒ new econometric model, the MDHL model, allowing us to empirically infer the presence of liquidity frictions...

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- Our specification gives some possible explanations:
 - Information is not the only source of price and volume variations;
 - Intradaily market characteristics do not convey the same information as daily market characteristics;
 - The total traded volume does not help identify the presence of liquidity frictions.

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 - The component of volume due to information: Information-revealing volume;
 - The component of volume due to liquidity shocks: Liquidity-adjusting volume;
 - New liquidity measure: $p\mu_v^{la}$.

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 - New liquidity measure: $p\mu_v^{la}$.
- The volume-volatility relation is positive but modified as compared to TP.

Further research

- The MDHL model provides a measure for average intraday liquidity shocks using daily data;
- \Rightarrow Confront it to liquidity microstructure measures;
- \Rightarrow Empirical tests of the validity of our liquidity measure.

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 - Price change and volume time series exhibit serial correlation;
- \Rightarrow The MDHL can be extended to allow for serial dependence in L_t .

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 - Price change and volume time series exhibit serial correlation;
- \Rightarrow The MDHL can be extended to allow for serial dependence in L_t .
 - Extraction of *I_t* and *L_t* time series from the observed return and volume data;
- \Rightarrow Stochastic volatility models with Markov regime change;
- \Rightarrow Kalman filter and simulation methods.

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Related projects

- Tracking illiquidities in daily and intradaily characteristics (working paper):
 - Short term liquidity frictions vs long lasting liquidity problems \Rightarrow different impacts on the (intra)daily volatility and volume time series;
 - Extended MDHL model and filtering techniques to extract latent liquidity factors from daily time series;
 - Implications for high (low) frequency arbitrage trading strategies.
- How to use the volatility-volume relation to measure liquidity (work in progress):
 - Confront MDHL-based liquidity measures to high frequency standard measures;
 - Compare relative performances of arbitrage strategies using different liquidity indicators;
 - Total traded volume versus liquidity-based volume...
- Build up market liquidity indicators (project):
 - Cross-sectional factor analysis to capture the essence of commonalities in liquidity shocks;